

Threshold corrections to rapidity distributions of Z and W^\pm bosons beyond N^2LO at hadron colliders

V. Ravindran¹, J. Smith².

¹*Harish-Chandra Research Institute, Chhatnag Road, Jhansi, Allahabad, India,*

²*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook,
NY 11794-3840 USA.*

ABSTRACT

Threshold enhanced perturbative QCD corrections to rapidity distributions of Z and W^\pm bosons at hadron colliders are presented using the Sudakov resummed cross sections at N^3LO level. We have used renormalisation group invariance and the mass factorisation theorem that these hard scattering cross sections satisfy to construct the QCD amplitudes. We show that these higher order threshold QCD corrections stabilise the theoretical predictions for vector boson production at the LHC under variations of both renormalisation and factorisation scales.

This paper is dedicated to the memory of W.L.G.A.M. van Neerven.

¹ravindra@mri.ernet.in

²smith@max2.physics.sunysb.edu

Recent theoretical advances in the computations of higher order radiative corrections in perturbative Quantum Chromodynamics (pQCD) have lead to extremely accurate predictions for several important observables needed for physics studies at the Tevatron collider in Fermilab as well as at the upcoming Large Hadron Collider (LHC) in CERN [1]. The Drell-Yan (DY) production of di-leptons [2], which is one of the dominant production processes at hadron colliders, can be used to precisely calibrate the experimental detectors. In addition, the DY process provides precise measurements of various standard model parameters through measurements of the rapidity distributions of Z bosons [3] and charge asymmetries of leptons coming from W boson decays [4]. Possible excess events in di-lepton invariant mass distributions can point to physics beyond the standard model, such as R -parity violating supersymmetric models, models with Z' , or with contact interactions [5] and gravity mediated models (see [6] for recent update). The precise measurements of Z and W boson production cross sections, various distributions and asymmetries by both the D0 and CDF collaborations [7] at the Fermilab Tevatron, where $\sqrt{S} = 1.96$ TeV, have already provided stringent tests of the standard model. These have already played an important role in bounding the mass of the Higgs boson. Similar measurements at the LHC will provide even more stringent tests due to the increase in the number of events at $\sqrt{S} = 14$ TeV.

The total cross sections for the Z and W^\pm production are known in pQCD up to next-to-next-to-leading order (N^2LO) [8–14]. Resummation programs for the threshold corrections to the total cross sections for DY production of di-leptons are also known [15, 16] (see also [17]) and one can consult [18, 19] for next-to-next-to-leading logarithmic (N^2LL) resummation results so it is straightforward to study the threshold effects in Z and W^\pm production at the cross section level. Recent QCD results at the three loop level [20–26] have lead to predictions for the resummation up to N^3LL [27–30]. Notice that the fixed order partial-soft-plus-virtual N^3LO corrections [27, 30] to the Higgs and DY production show the reliability of the perturbation theory results and demonstrate stability against the variations of renormalization and mass factorization scales. Exact results up to N^2LO are also available for less inclusive observables for di-lepton [31], Z and W^\pm [32] (see [33–35]) production. Recently the dominant QCD threshold corrections to the rapidity distribution of di-leptons in the DY process at N^3LO have been obtained in [36]. It was found that these corrections are indeed small and reduce the scale uncertainties significantly making the predictions more reliable. The fixed order results as well as the resummed results reveal very interesting structures in the perturbative QCD series (see, [37–42]).

The hard scattering cross sections computed using the QCD improved parton model are often sensitive to variations in the renormalisation and factorisation scales usually denoted by μ_R and μ_F respectively. The former originate from ultraviolet renormalisation while the latter originate in the mass factorisation of collinear singularities. In addition to the scale uncertainties the fixed order computations suffer from the presence of various large logarithms which arise in some kinematical regions. These regions are often important from the experimental point of view and these large logarithms, which spoil the standard perturbative predictions, should be resummed in a closed form. For instance resummation formulae supplemented with fixed order results can predict the dominant higher order threshold corrections to various observables. These threshold corrections are large when the fluxes of the incoming partons are large, which occurs at large partonic energies.

In [37] we computed the soft distribution functions that resum the soft gluons coming from real gluon emission processes in DY production and Higgs production and also found that they are related by the colour factor C_A/C_F , see also [43] and [44]. Using the soft distribution functions extracted from DY, and the form factor of the Yukawa coupling of Higgs to bottom quarks, we predicted the soft-plus-virtual (sv) parts of the Higgs production cross section through bottom quark annihilation beyond N²LO with the same accuracy that the DY process and the gluon fusion to Higgs process are known [27, 30]. This approach was then successfully applied in [38] to Higgs decay to bottom quarks and hadroproduction in e^+e^- annihilation. Since our results in [30, 37, 38] are related to that of the standard threshold resummation, we could determine [37] the threshold exponents D_i^I up to three loop level for DY and Higgs production using our resummed soft distribution functions and the quantities B_i^I for deep inelastic scattering, Higgs decay and the hadroproduction of Higgs bosons. In [36] we extended this approach to include x_F and rapidity differential cross sections for di-lepton pairs in DY production and for Higgs bosons in Higgs production processes. In this paper we apply these same methods to study the effects of the dominant threshold corrections at N³LO to the rapidity distributions of the Z and W^\pm bosons in hadron-hadron collisions.

In [36] we formulated a framework to resum the dominant soft gluon contributions coming from the threshold region to the x_F and rapidity distributions of DY di-lepton pairs and Higgs bosons at hadron colliders in the z_i ($i = 1, 2$) space of the kinematic variables. We recapitulate the main points here to make this paper more understandable. The threshold region corresponds to $z_i \rightarrow 1$ and in this region all the partonic cross sections are symmetric in $z_1 \leftrightarrow z_2$. To obtain the resummed result, we used renormalisation group (RG) invariance, mass factorisation and Sudakov resummation of QCD amplitudes. Using the resummed results in z_i space we predicted the sv parts (also called threshold corrections) of the dominant partonic x_F and rapidity distributions beyond N²LO. We follow the similar approach here to obtain the dominant threshold corrections at N³LO level for the rapidity distributions of Z and W^\pm bosons at both the LHC and Tevatron energies. See [45] for an early reference where the resummation for DY differential distributions at rapidity $Y = 0$ (or $x_F = 0$) was considered.

The differential cross section for producing a vector boson can be expressed as:

$$\frac{d^2\sigma^J}{dq^2 dy} = \sigma_{\text{Born}}^J(x_1^0, x_2^0, q^2) W^I(x_1^0, x_2^0, q^2), \quad (1)$$

where q is the four-momentum of the vector boson. In our case $q^2 = M_J^2$ where $J = Z, W^\pm$ but for convenience we use q^2 for most of this paper. Later we will present plots for $d^2\sigma^J/dq^2 dy$, where q now represents $\sqrt{q^2} = M_Z$ or M_W for Z and W^\pm respectively. Our normalisation is $W_{\text{Born}}^I(x_1^0, x_2^0, q^2) = \delta(1 - x_1^0)\delta(1 - x_2^0)$. The superscript I represents light-quarks (q), gluons (g) and heavy quarks (b) but we only need the first case here so $I = q$ for the rest of the paper. The x_i^0 ($i = 1, 2$) are related to q^2 , the scaling variable $\tau = q^2/S$, and the rapidity y of the vector boson J :

$$y = \frac{1}{2} \log \left(\frac{p_2 \cdot q}{p_1 \cdot q} \right) = \frac{1}{2} \log \left(\frac{x_1^0}{x_2^0} \right), \quad \tau = x_1^0 x_2^0. \quad (2)$$

Here $S = (p_1 + p_2)^2$ is the square of the hadronic center of mass energy and p_i are the momenta of incoming hadrons P_i ($i = 1, 2$).

In the QCD improved parton model, the function $W^I(x_1^0, x_2^0, q^2)$ can be expressed in terms of the fitted parton distribution functions (PDFs) appropriately convoluted with perturbatively calculable partonic differential cross sections denoted by $\Delta_{d,ab}^I$ as follows

$$W^I(x_1^0, x_2^0, q^2) = \sum_{a,b=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2 \mathcal{H}_{ab}^I(x_1, x_2, \mu_F^2) \times \int_0^1 dz_1 \int_0^1 dz_2 \delta(x_1^0 - x_1 z_1) \delta(x_2^0 - x_2 z_2) \Delta_{d,ab}^I(z_1, z_2, q^2, \mu_F^2, \mu_R^2), \quad (3)$$

where the subscript d denotes the particular differential distribution one is studying (y , x_F etc). Here μ_R is the renormalisation scale and μ_F the factorisation scale. The function $\mathcal{H}_{ab}^I(x_1, x_2, \mu_F^2)$ is the product of PDFs $f_a(x_1, \mu_F^2)$ and $f_b(x_2, \mu_F^2)$ renormalised at the factorisation scale μ_F . That is,

$$\mathcal{H}_{ab}^q(x_1, x_2, \mu_F^2) = f_a^{P_1}(x_1, \mu_F^2) f_b^{P_2}(x_2, \mu_F^2), \quad (4)$$

with x_i ($i = 1, 2$) the momentum fractions of the partons in the incoming hadrons.

The partonic cross sections can be expressed in terms of soft and hard parts. The soft parts come from the soft gluons that appear in real emission as well as in the virtual processes. The infra-red safe contributions from the soft gluons can be obtained by adding the soft parts of the differential cross sections with the ultraviolet renormalised virtual contributions and performing mass factorisation using appropriate counter terms. These combinations are called the "soft-plus-virtual" (sv) parts of the differential cross sections. Hence we write

$$\Delta_{d,ab}^I(z_1, z_2, q^2, \mu_F^2, \mu_R^2) = \Delta_{d,ab}^{\text{hard}}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) + \delta_{ab} \Delta_{d,I}^{\text{sv}}(z_1, z_2, q^2, \mu_F^2, \mu_R^2), \quad I = q. \quad (5)$$

The hard parts of the differential cross sections $\Delta_{d,ab}^{\text{hard}}(z_1, z_2, q^2, \mu_F^2, \mu_R^2)$ can be obtained by the standard procedure(see [6, 46]). The sv parts of the differential cross sections are obtained using the method discussed in the [36] so that

$$\Delta_{d,I}^{\text{sv}}(z_1, z_2, q^2, \mu_R^2, \mu_F^2) = C \exp \left(\Psi_d^I(q^2, \mu_R^2, \mu_F^2, z_1, z_2, \epsilon) \right) \Big|_{\epsilon=0}, \quad (6)$$

where the $\Psi_d^I(q^2, \mu_R^2, \mu_F^2, z_1, z_2, \epsilon)$ are finite distributions computed in $4 + \epsilon$ dimensions and they take the form

$$\begin{aligned} \Psi_d^I(q^2, \mu_R^2, \mu_F^2, z_1, z_2, \epsilon) = & \left(\ln |\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)|^2 \right) \delta(1 - z_1) \delta(1 - z_2) \\ & + 2 \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) - C \ln \Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, z_1, \epsilon) \delta(1 - z_2) \\ & - C \ln \Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, z_2, \epsilon) \delta(1 - z_1). \end{aligned} \quad (7)$$

The symbol " \mathcal{C} " means convolution. For example, \mathcal{C} acting on the exponential of a function $f(z_1, z_2)$ means the following expansion:

$$\begin{aligned} \mathcal{C} e^{f(z_1, z_2)} &= \delta(1-z_1)\delta(1-z_2) + \frac{1}{1!}f(z_1, z_2) + \frac{1}{2!}f(z_1, z_2) \otimes f(z_1, z_2) \\ &+ \frac{1}{3!}f(z_1, z_2) \otimes f(z_1, z_2) \otimes f(z_1, z_2) + \dots \end{aligned} \quad (8)$$

In the rest of the paper the function $f(z_1, z_2)$ is a distribution of the kind $\delta(1-z_j)$ or $\mathcal{D}_i(z_j)$, where

$$\mathcal{D}_i(z_j) = \left[\frac{\ln^i(1-z_j)}{(1-z_j)} \right]_+ \quad i = 0, 1, \dots, \quad \text{and} \quad j = 1, 2, \quad (9)$$

and the symbol \otimes means the "double" Mellin convolution with respect to the variables z_1 and z_2 . We drop all the regular functions that result from these convolutions when defining the sv part of the cross sections. The $\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)$ are the standard form factors coming from the purely virtual parts of the cross sections. In the form factors, we have $Q^2 = -M_J^2$. The partonic cross sections depend on two scaling variables z_1 and z_2 . The functions $\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon)$ are called the soft distribution functions. The unrenormalised (bare) strong coupling constant \hat{a}_s is defined as

$$\hat{a}_s = \frac{\hat{g}_s^2}{16\pi^2}, \quad (10)$$

where \hat{g}_s is the strong coupling constant which is dimensionless in $n = 4 + \epsilon$ space time dimensions. The scale μ comes from dimensional regularisation which makes the bare coupling constant \hat{g}_s dimensionless in n dimensions. The bare coupling constant \hat{a}_s is related to renormalised one by the following relation:

$$S_\epsilon \hat{a}_s = Z(\mu_R^2) a_s(\mu_R^2) \left(\frac{\mu^2}{\mu_R^2} \right)^{\frac{\epsilon}{2}}, \quad (11)$$

where $S_\epsilon = \exp \left\{ \frac{\epsilon}{2} [\gamma_E - \ln 4\pi] \right\}$ is the spherical factor characteristic of n -dimensional regularisation. The renormalisation constant $Z(\mu_R^2)$ relates the bare coupling constant \hat{a}_s to the renormalised one $a_s(\mu_R^2)$. They are both expressed in terms of the perturbatively calculable coefficients β_i which are known up to four-loop level [47, 48] in terms of the colour factors of SU(N) gauge group:

$$C_A = N, \quad C_F = \frac{N^2 - 1}{2N}, \quad T_F = \frac{1}{2}. \quad (12)$$

Also we use n_f for the number of active flavours.

In dimensional regularisation, the bare form factors $\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)$ satisfy the following differential equation [49–52].

$$Q^2 \frac{d}{dQ^2} \ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon) = \frac{1}{2} \left[K^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon \right) + G^I \left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon \right) \right]. \quad (13)$$

The fact that the $\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)$ are renormalisation group invariant and the functions G^I are finite implies that the K^I terms can be expressed in terms of finite constants A^I , the so-called cusp anomalous dimensions and the coefficients β_i . The formal solution to the eqn.(13), in dimensional regularisation, up to four-loop level is obtained in [22,37]. The finite constants $G_i^I(\epsilon)$ (see eqn.(19) of [30]) are also known [23] to the required accuracy in ϵ . These constants $G_i^I(\epsilon)$ are expressed in terms of the functions B_i^I and f_i^I . The B_i^I are known up to order a_s^3 through the three-loop anomalous dimensions (or splitting functions) [20, 21] and are found to be flavour independent, that is $B_i^q = B_i^b$. The constants f_i^I are analogous to the cusp anomalous dimensions A_i^I that enter the form factors with $A_i^q = A_i^b$. It was first noticed in [43] that the single pole terms in ϵ in the logarithms of the quark and gluon form factors up to two-loop level (a_s^2) can be predicted by the $C_F \rightarrow C_A$ substitution. The structure of single pole terms of four-point amplitudes at the two-loop level can be found in [53, 54]. The UV divergences present in the form factor are removed when the bare coupling constant \hat{a}_s undergoes renormalisation via the eqn.(11).

The collinear singularities that arise due to the presence of massless partons are removed using the mass factorisation kernels $\Gamma(z_j, \mu_F^2, \epsilon)$ in the $\overline{\text{MS}}$ scheme (see eqn.(7)). We suppress their dependence on \hat{a}_s and μ^2 . The factorisation kernels $\Gamma(z_j, \mu_F^2, \epsilon)$ satisfy the following renormalisation group equations:

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma(z_j, \mu_F^2, \epsilon) = \frac{1}{2} P(z_j, \mu_F^2) \otimes \Gamma(z_j, \mu_F^2, \epsilon) \quad j = 1, 2, \quad (14)$$

where the $P(z_j, \mu_F^2)$ are the DGLAP matrix-valued splitting functions which are known up to three-loop level [20, 21]:

$$P(z_j, \mu_F^2) = \sum_{i=1}^{\infty} a_s^i(\mu_F^2) P^{(i-1)}(z_j). \quad (15)$$

The diagonal terms in the splitting functions $P^{(i)}(z_j)$ have the following structure

$$P_{II}^{(i)}(z_j) = 2 \left[B_{i+1}^I \delta(1 - z_j) + A_{i+1}^I \mathcal{D}_0(z_j) \right] + P_{\text{reg}, II}^{(i)}(z_j), \quad (16)$$

where $P_{\text{reg}, II}^{(i)}(z_j)$ are regular when the argument approaches the kinematic limit (here $z_j \rightarrow 1$). The RG equations can be solved by expanding them in powers of the strong coupling constant. Only the diagonal parts of the kernels contribute to the sv parts of the differential cross sections. We find the solutions contain only poles in ϵ in the $\overline{\text{MS}}$ scheme:

$$\Gamma(z_j, \mu_F^2, \epsilon) = \delta(1 - z_j) + \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_F^2}{\mu^2} \right)^{i \frac{\epsilon}{2}} S_{\epsilon}^i \Gamma^{(i)}(z_j, \epsilon). \quad (17)$$

An expansion for the $\Gamma^{(i)}(z_j, \epsilon)$ in negative powers of ϵ up to four-loop level can be found in [37]. The $\Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, z_j, \epsilon)$ in eqn.(7) is the diagonal element of $\Gamma(z_j, \mu_F^2, \epsilon)$.

From the eqn.(13) and the fact that the $\Delta_{d, I}^{\text{sv}}$ are finite in the limit $\epsilon \rightarrow 0$ we obtain

$$q^2 \frac{d}{dq^2} \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = \frac{1}{2} \left[\overline{K}_d^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \epsilon \right) + \overline{G}_d^I \left(\hat{a}_s, \frac{q^2}{\mu^2}, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \epsilon \right) \right], \quad (18)$$

where now the constants \overline{K}_d^I contain all the singular terms in ϵ and the \overline{G}_d^I are finite functions of ϵ . The functions $\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon)$ also satisfy the renormalisation group equations:

$$\mu_R^2 \frac{d}{d\mu_R^2} \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = 0. \quad (19)$$

The $\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon)$ should contain the correct poles to cancel the poles coming from \hat{F}^I, Z^I and Γ_{II} in order to make $\Delta_{d,I}^{\text{sv}}$ finite. This requirement unambiguously determines all the poles of this distribution. The solution to the Sudakov differential equation for the soft distribution functions in eqn.(18) can be written as

$$\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i S_{\epsilon}^i \left(\frac{q^2(1-z_1)(1-z_2)}{\mu^2} \right)^{i\frac{\epsilon}{2}} \left(\frac{(i\epsilon)^2}{4(1-z_1)(1-z_2)} \right) \hat{\phi}_d^{I,(i)}(\epsilon) \quad (20)$$

where

$$\hat{\phi}_d^{I,(i)}(\epsilon) = \frac{1}{i\epsilon} \left[\overline{K}_d^{I,(i)}(\epsilon) + \overline{G}_d^{I,(i)}(\epsilon) \right]. \quad (21)$$

The constants $\overline{K}_d^{I,(i)}(\epsilon)$ are expanded in powers of the bare coupling constant \hat{a}_s as follows

$$\overline{K}_d^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \epsilon \right) = \delta(1-z_1)\delta(1-z_2) \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2} \right)^{i\frac{\epsilon}{2}} S_{\epsilon}^i \overline{K}_d^{I,(i)}(\epsilon). \quad (22)$$

Using the RG equation for $\overline{K}_d^I(\hat{a}_s, \mu_R^2/\mu^2, z_1, z_2, \epsilon)$, one finds that the constants $\overline{K}_d^{I,(i)}(\epsilon)$ are identical to $\overline{K}^{I,(i)}(\epsilon)$ given in [30]. The constants $\overline{G}_d^{I,(i)}(\epsilon)$ are related to the finite boundary functions $\overline{G}_d^I(a_s(q^2), 1, z_1, z_2, \epsilon)$. We define the $\overline{g}_{d,i}^I(\epsilon)$ through the relation

$$\sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z_1)(1-z_2)}{\mu^2} \right)^{i\frac{\epsilon}{2}} S_{\epsilon}^i \overline{G}_d^{I,(i)}(\epsilon) = \sum_{i=1}^{\infty} a_s^i (q^2(1-z_1)(1-z_2)) \overline{g}_{d,i}^I(\epsilon). \quad (23)$$

We obtain the z_1, z_2 independent constants $\overline{g}_{d,i}^I(\epsilon)$ by demanding the finiteness of $\Delta_{d,I}^{\text{sv}}$ given in eqn.(6). Before setting $\epsilon = 0$ in eqn.(6), we expand $\Delta_{d,I}^{\text{sv}}$ as

$$\Delta_{d,I}^{\text{sv}}(z_1, z_2, q^2, \mu_R^2, \mu_F^2, \epsilon) = \sum_{i=0}^{\infty} a_s^i(\mu_R^2) \Delta_{d,I}^{\text{sv},(i)}(z_1, z_2, q^2, \mu_R^2, \mu_F^2, \epsilon). \quad (24)$$

Using the above expansion and eqn.(7) we determine these constants using the known information on the form factors, the mass factorisation kernels and the coefficient functions $\Delta_{d,I}^{\text{sv},(i-1)}$ expanded in powers of ϵ . The structure of the $G_d^I(\epsilon)$ in the form factors involving the constants f^I and β_i was given in [30]. The constants $\overline{g}_{d,i}^I(\epsilon)$ in the soft distribution functions also have a similar structure:

$$\overline{g}_{d,1}^I(\epsilon) = -f_1^I + \sum_{k=1}^{\infty} \epsilon^k \overline{g}_{d,1}^{I,(k)},$$

$$\begin{aligned}
\overline{\mathcal{G}}_{d,2}^I(\epsilon) &= -f_2^I - 2\beta_0 \overline{\mathcal{G}}_{d,1}^{I,(1)} + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}_{d,2}^{I,(k)}, \\
\overline{\mathcal{G}}_{d,3}^I(\epsilon) &= -f_3^I - 2\beta_1 \overline{\mathcal{G}}_{d,1}^{I,(1)} - 2\beta_0 \left(\overline{\mathcal{G}}_{d,2}^{I,(1)} + 2\beta_0 \overline{\mathcal{G}}_{d,1}^{I,(2)} \right) + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}_{d,3}^{I,(k)}, \\
\overline{\mathcal{G}}_{d,4}^I(\epsilon) &= -f_4^I - 2\beta_2 \overline{\mathcal{G}}_{d,1}^{I,(1)} - 2\beta_1 \left(\overline{\mathcal{G}}_{d,2}^{I,(1)} + 4\beta_0 \overline{\mathcal{G}}_{d,1}^{I,(2)} \right), \\
&\quad - 2\beta_0 \left(\overline{\mathcal{G}}_{d,3}^{I,(1)} + 2\beta_0 \overline{\mathcal{G}}_{d,2}^{I,(2)} + 4\beta_0^2 \overline{\mathcal{G}}_{d,1}^{I,(3)} \right) + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}_{d,4}^{I,(k)}. \tag{25}
\end{aligned}$$

The terms proportional to ϵ at every order in \hat{a}_s are determined using the known σ^J to order N²LO and the following identity:

$$\int_0^1 dx_1^0 \int_0^1 dx_2^0 (x_1^0 x_2^0)^{N-1} \frac{d\sigma^J}{dy} = \int_0^1 d\tau \tau^{N-1} \sigma^J. \tag{26}$$

We find

$$\begin{aligned}
\overline{\mathcal{G}}_{d,1}^{q,(1)} &= C_F \left(-\zeta_2 \right), \\
\overline{\mathcal{G}}_{d,1}^{q,(2)} &= C_F \left(\frac{1}{3} \zeta_3 \right), \\
\overline{\mathcal{G}}_{d,1}^{q,(3)} &= C_F \left(\frac{1}{80} \zeta_2^2 \right), \\
\overline{\mathcal{G}}_{d,2}^{q,(1)} &= C_F C_A \left(\frac{2428}{81} - \frac{67}{3} \zeta_2 - 4\zeta_2^2 - \frac{44}{3} \zeta_3 \right) \\
&\quad + C_F n_f \left(-\frac{328}{81} + \frac{10}{3} \zeta_2 + \frac{8}{3} \zeta_3 \right). \tag{27}
\end{aligned}$$

Using the resummed result given in eqn.(6), the exponents $g_i^q(\epsilon)$ (see [23]) and $\overline{\mathcal{G}}_{d,i}^q(\epsilon)$, we can obtain the higher order sv contributions to the differential cross sections. The available exponents are

$$\begin{aligned}
g_1^{q,j}, \quad \overline{\mathcal{G}}_{d,1}^{q,(j)} &\quad \text{for} \quad j = \text{all} \quad , \\
g_2^{q,j}, \quad \overline{\mathcal{G}}_{d,2}^{q,(j)} &\quad \text{for} \quad j = 0, 1 \quad , \\
g_3^{q,j}, \quad \overline{\mathcal{G}}_{d,3}^{q,(j)} &\quad \text{for} \quad j = 0 \quad ,
\end{aligned}$$

in addition to the known β_i ($i = 0, 1, 2, 3$) and the constants in the splitting functions A_i^q, B_i^q ($i = 1, 2, 3$) and f_i^q ($i = 1, 2, 3$). The constants $g_2^{q,j}$ are known for $j = 2, 3$ also (see [22]). Using our

approach we have obtained the exact $\Delta_{d,q}^{\text{sv},(i)}$ up to N²LO ($i = 0, 1, 2$) [32]. The coefficient of the $\delta(1 - z_1)\delta(1 - z_2)$ part depends on the constants $\overline{g}_2^{q,(2)}, g_3^{q,1}, \overline{g}_3^{q,(1)}$ which are still unknown for N³LO, so we can only obtain a *partial result* for $\Delta_{d,q}^{\text{sv},(3)}$, i.e., a result without the $\delta(1 - z_1)\delta(1 - z_2)$ part can be computed from our formula given in eqn.(6). We can also obtain a result to N⁴LO order where we can predict *partial* sv contributions containing everything except the terms in $\mathcal{D}_0(z_i)\delta(1 - z_j), \mathcal{D}_0(z_i)\mathcal{D}_0(z_j), \mathcal{D}_1(z_i)\delta(1 - z_j)$ and $\delta(1 - z_1)\delta(1 - z_2)$ for the coefficient $\Delta_{d,q}^{\text{sv},(4)}$. The results are identical to those given in the Appendix B of [36] for $\mu_R^2 = \mu_F^2 = M_Z^2$. The convolutions of distributions of the form $\mathcal{D}_l(z_j) \otimes \mathcal{D}_m(z_j)$ for any arbitrary l, m can be done using the general formulae given in [30] so we obtain $\Delta_{d,I}^{\text{sv},(i)}$ for $i = 1, \dots, 4$. The differential cross sections

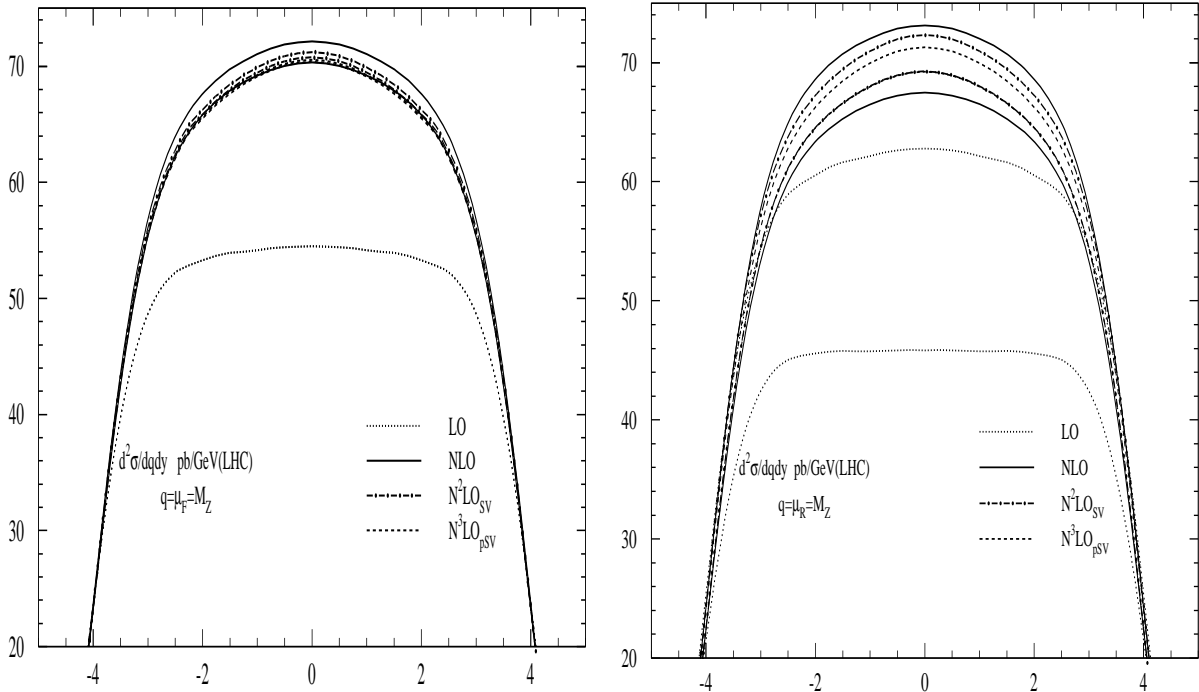


Figure 1: Rapidity distributions for Z boson production at the LHC, and their $\mu = \mu_R$ (left panel) and $\mu = \mu_F$ (right panel) scale dependence (with $M_Z^2/2 < \mu^2 < 2M_Z^2$). The abbreviation "pSV" means partial-soft-plus-virtual.

can be expanded in powers of the strong coupling constant as

$$\frac{d^2 \sigma^J}{dq^2 dy} = \sum_{i=0}^{\infty} a_s^i \frac{d^2 \sigma^{J,(i)}}{dq^2 dy}. \quad (28)$$

We split the partonic cross section into hard and sv parts:

$$\frac{d^2 \sigma^{J,(i)}}{dq^2 dy} = \frac{d^2 \sigma^{\text{hard},J,(i)}}{dq^2 dy} + \frac{d^2 \sigma^{\text{sv},J,(i)}}{dq^2 dy}, \quad (29)$$

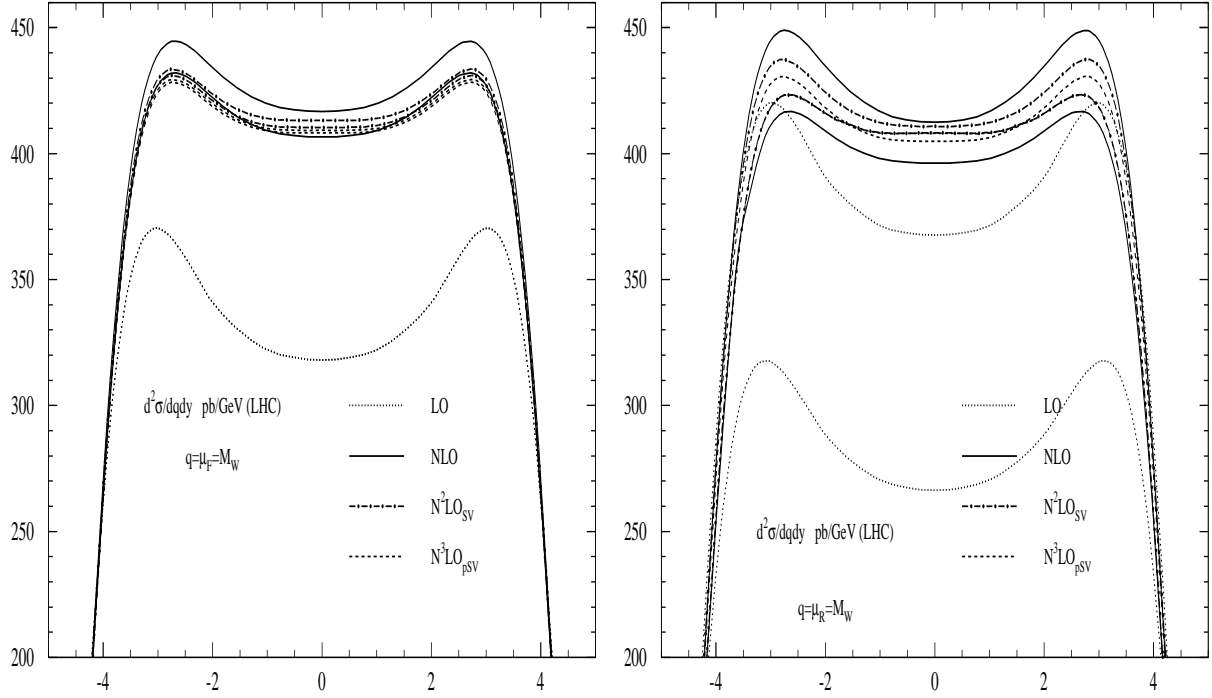


Figure 2: Rapidity distributions for W^+ boson production at the LHC, and their $\mu = \mu_R$ (left panel) and $\mu = \mu_F$ (right panel) scale dependence (with $M_W^2/2 < \mu^2 < 2M_W^2$). The abbreviation "pSV" means partial-soft-plus-virtual.

$$2S \frac{d^2 \sigma^{\text{hard}, J, (i)}}{dq^2 dy} = \sum_q \mathcal{G}_{SM}^J \left(D_{q\bar{q}}^{SM, (i)}(x_1^0, x_2^0, \mu_F^2) + D_{qg}^{SM, (i)}(x_1^0, x_2^0, \mu_F^2) + D_{gq}^{SM, (i)}(x_1^0, x_2^0, \mu_F^2) \right). \quad (30)$$

The SM coefficients $D_{ab}^{SM, (i)}(x_1^0, x_2^0, \mu_F^2)$ can be found in [6, 46]. The sv part of the partonic cross section can be expressed as

$$2S \frac{d^2 \sigma^{\text{sv}, J, (i)}}{dq^2 dy} = \sum_{a, b=q, \bar{q}} \mathcal{G}_{SM}^J \int_0^1 dx_1 \int_0^1 dx_2 \mathcal{H}_{ab}^q(x_1, x_2, \mu_F^2) \times \int_0^1 dz_1 \int_0^1 dz_2 \delta(x_1^0 - x_1 z_1) \delta(x_2^0 - x_2 z_2) \Delta_{d, q}^{\text{sv}, (i)}(z_1, z_2, q^2, \mu_F^2, \mu_R^2). \quad (31)$$

The coefficients $\Delta_{d, ab}^{\text{sv}, (i)}(z_1, z_2, q^2, \mu_F^2, \mu_R^2)$ are presented in the Appendix B of [36], with the normalisation $\Delta_{y, ab}^{\text{sv}, (0)}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) = \delta(1 - z_1)\delta(1 - z_2)$. The functions \mathcal{G}_{SM}^J are given by

$$\mathcal{G}_{SM}^{M_Z} = \frac{4\alpha^2 Q_q^2}{3q^2} + \frac{4\alpha q^2 \Gamma_{Z \rightarrow l+l^-}}{M_Z ((q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2) c_w^2 s_w^2} \left((g_q^V)^2 + (g_q^A)^2 \right)$$

$$+ \frac{2\alpha^2(1-4s_w^2)(q^2-M_Z^2)}{3((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)c_w^2s_w^2} Q_q g_q^V, \quad (32)$$

$$\mathcal{G}_{SM}^{M_W} = \frac{\alpha q^2 \Gamma_{W \rightarrow l \bar{\nu}_l}}{((q^2-M_W^2)^2+M_W^2\Gamma_W^2) M_W s_w^2} V_{ij}^2. \quad (33)$$

We now give results for Z and W^\pm production by choosing $q^2 = M_Z^2$ and $q^2 = M_W^2$ respectively. At these points the functions above contain the standard electro-weak constants which can be found in [6, 44] and the standard CKM matrix elements V_{ij} . We present our results as differential cross sections in rapidity for these fixed q^2 values and to compare with other authors plot $d^2\sigma/dqdy$ where $q = M_Z$ or $q = M_W$ respectively.

We choose the center-of-mass energy to be $\sqrt{S}=14$ TeV for the LHC and $\sqrt{S}=1.96$ TeV for the Tevatron. The Z boson mass is taken to be $M_Z = 91.19$ GeV and the width is 2.50 GeV. The corresponding values for the W boson are $M_W = 80.43$ and 2.12 GeV respectively. The strong coupling constant $\alpha_s(\mu_R^2)$ is evolved using the 4-loop RG equations depending on the order in which the cross section is evaluated. We choose $\alpha_s^{\text{LO}}(M_Z) = 0.130$, $\alpha_s^{\text{NLO}}(M_Z) = 0.119$, $\alpha_s^{\text{N}^2\text{LO}}(M_Z) = 0.115$ and $\alpha_s^{\text{N}^3\text{LO}}(M_Z) = 0.113$ for $i > 2$. The set MRST 2001 LO is used for leading order, MRST2001 NLO for NLO and MRST 2002 NNLO for N^iLO with $i > 1$ [55, 56]. By choosing these parton densities we can compare our results with those of other authors (see later). We use $\alpha = 1/128$ for the electromagnetic fine structure constant, $\sin^2\theta_W = 0.2314$ for the weak mixing angle and $\cos^2\theta_C = 0.975$ for the Cabibbo angle. In fig. 1 we plot the rapidity distributions for the Z -boson at the LHC in LO (dotted lines), NLO (solid lines), N^2LO (SV only, dot-dashed lines) and N^3LO (pSV only, small-dashed lines). Note that we have not plotted the partial sv N^4LO contributions and we have only plotted the curves above 20 pb/GeV to magnify the central rapidity region. There are two panels in this plot and two curves in each panel since we show the scale variations by varying the mass factorization scale μ_F in the parton densities and the mass renormalization scale μ_R in the coefficient functions. Therefore we plot the curves at fixed $\mu_F^2 = M_Z^2$ but with $\mu_R^2 = M_Z^2/2$ and $\mu_R^2 = 2M_Z^2$ in the left panel and fixed $\mu_R^2 = M_Z^2$ but with $\mu_F^2 = M_Z^2/2$ and $\mu_F^2 = 2M_Z^2$ in the right panel. Our results in the left panel show that there is only a tiny dependence on the μ_R for fixed μ_F (here the LO result has no variation). In the right panel we see that μ_F dependence for fixed μ_R decreases as we go from LO to NLO, N^2LO and N^3LO respectively. The N^3LO band lies within the N^2LO band and both are within the bands for the NLO results. We also notice that the lower curves for the N^2LO and N^3LO results fall on top of each other. The actual numbers are different but so close that one cannot see this from the plot. These results demonstrate that the perturbation series for the rapidity distribution converges very nicely at the LHC energy.

In fig. 2 we plot the rapidity distributions for the W^+ -boson at the LHC in LO (dotted lines), NLO (solid lines), N^2LO (SV only, dot-dashed lines) and N^3LO (pSV only, small-dashed lines). We have only plotted the curves above 200 pb/GeV to magnify the central rapidity region. Again there are two panels in this plot and two curves in each panel since we show the scale variations by

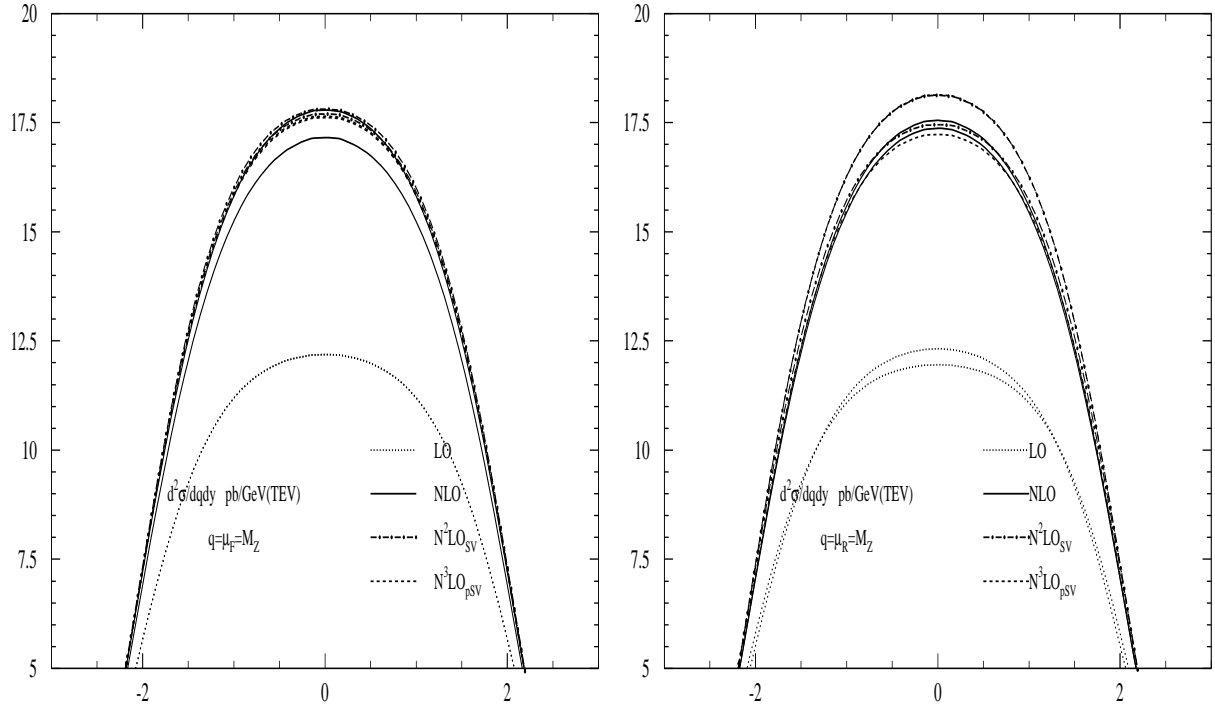


Figure 3: Rapidity distributions for Z boson production at the Tevatron, and their $\mu = \mu_R$ (left panel) and $\mu = \mu_F$ (right panel) scale dependence (with $M_Z^2/2 < \mu^2 < 2M_Z^2$). The abbreviation "pSV" means partial-soft-plus-virtual.

varying the mass factorization scale μ_F in the parton densities and the mass renormalization scale μ_R in the coefficient functions. Therefore we plot the curves at fixed $\mu_F^2 = M_W^2$ but with $\mu_R^2 = M_W^2/2$ and $\mu_R^2 = 2M_W^2$ in the left panel and fixed $\mu_R^2 = M_W^2$ but with $\mu_F^2 = M_W^2/2$ and $\mu_F^2 = 2M_W^2$ in the right panel. Our results in the left panel show that there is only a tiny dependence on the μ_R for fixed μ_F (here the LO result has no variation). In the right panel we see that μ_F dependence for fixed μ_R decreases as we go from LO to NLO, $N^2\text{LO}$ and $N^3\text{LO}$ respectively. However the $N^3\text{LO}$ band is slightly below the $N^2\text{LO}$ band near $y = 0$ even though both are within the bands for the NLO results, which is probably caused by the fact that we only have a partial soft-plus-virtual $N^3\text{LO}$ result. We notice again that the lower curves for the $N^2\text{LO}$ and $N^3\text{LO}$ results in the right panel fall on top of each other. The actual numbers are different but so close that one cannot see this from the plot. Both plots indicate that the perturbation series is rapidly converging at the LHC energy.

In figures 3 and 4 we repeat these plots for the Tevatron energy and the same scale choices as above. The left panel in Fig. 3 shows excellent convergence of our results. Note that we only plot our results above 5 pb/GeV to magnify the central rapidity region. However the right panel shows that the bands for the $N^3\text{LO}$ result are wider than those for the $N^2\text{LO}$ result and both are wider than those for the NLO result. This can have two reasons. One is that our results are only sv or partial sv. The other is that since the Tevatron is an antiproton-proton collider different combinations of parton densities are involved as we increase the order of the perturbation series. We discuss this

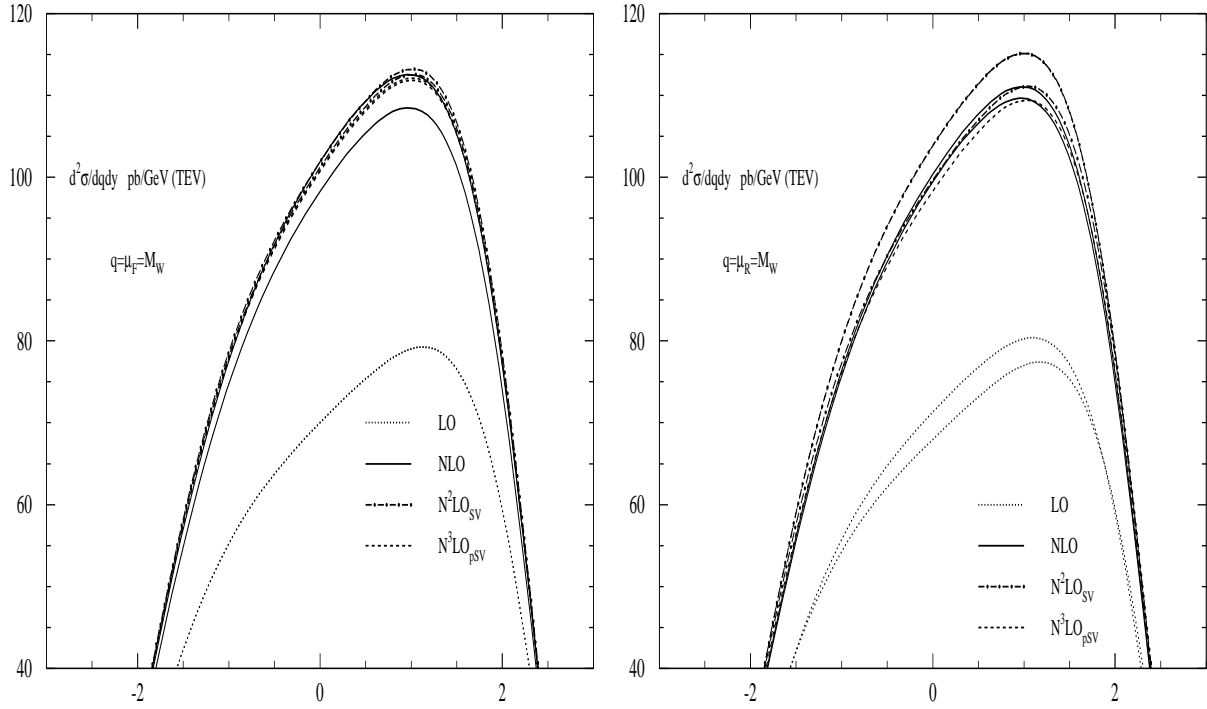


Figure 4: Rapidity distributions for W^+ boson production at the Tevatron, and their $\mu = \mu_R$ (left panel) and $\mu = \mu_F$ (right panel) scale dependence (with $M_W^2/2 < \mu^2 < 2M_W^2$). The abbreviation "pSV" means partial-soft-plus-virtual.

again below. We notice again that the lower curves for the $N^2\text{LO}$ and $N^3\text{LO}$ results in the right panel fall on top of each other. The actual numbers are different but so close that one cannot see this from the plot.

Figure 4 shows results for W^+ production above 40 pb/GeV to concentrate on the central rapidity region. The curves in the left panel show excellent convergence of the perturbation series. The right panel again shows that the bands for the $N^3\text{LO}$ result are wider than those for the $N^2\text{LO}$ result and both are wider than those for the NLO result. Again we believe that this is caused by a combination of the reasons above and comment on it below. We notice again that the lower curves for the $N^2\text{LO}$ and $N^3\text{LO}$ results in the right panel fall on top of each other. The actual numbers are different but so close that one cannot see this from the plot.

Note that the asymmetry about $y = 0$ in the rapidity plots for W^+ production at the Tevatron in Fig.4 has basically disappeared at the LHC energy (see Fig.2).

We have checked our results in two ways. First by comparing our curves with similar plots for the rapidity distributions in [12]. Their computer program for the rapidity distributions has the exact LO result, the exact NLO result and a sv approximation for the $N^2\text{LO}$ result. We agree with their numbers when we choose their values for the electroweak parameters and their parton densities. Second we have also checked our results against those in [32], where the exact $N^2\text{LO}$

rapidity distributions for the Z and W^\pm bosons are calculated. Their paper contains plots for the exact LO, the exact NLO and the exact N^2 LO results in pQCD. We have run their computer code to compare their results against ours. Our sv approximation agrees very well with their N^2 LO results in the case when $\mu_F = \mu_R = M_J/2$ for the Z and W^\pm bosons. However we get a slightly wider band when we vary the scales in the N^2 LO case, since we only have a sv approximation. From this comparison we can see that the wider bands we observe in Figures 3 and 4 are basically due to the sv and partial sv nature of our higher order results. We can probably reduce the width of the bands in the N^3 LO case by calculating the missing pieces of the partial sv result. However the central value of our partial sv N^3 LO result is very small when compared with the N^2 LO result indicating that the perturbation series continues to converge rather rapidly. Hence this is good news for the LHC experimenters who plan to calibrate the Atlas and CMS detectors by measuring the rapidity and transverse momentum distributions of Z and W^\pm bosons.

To summarise, we have systematically studied higher order sv corrections to rapidity differential distributions for Z and W^\pm boson production. We have used Sudakov resummation of soft gluons to calculate these processes. The resummation of soft gluons has been achieved using renormalisation group invariance and the factorisation property of the observable that is considered here (the rapidity). Using the available information on the form factors, the DGLAP kernels and lower order results we have obtained compact expressions for the resummation of soft gluons for the rapidity distributions of Z and W^\pm bosons. Using these we have computed sv rapidity distributions exactly at N^2 LO and partially at N^3 LO. We have presented the numerical impact of these results.

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